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# PROBABILISTIC SEASONAL-TREND DECOMPOSITION OF TIME SERIES USING LOCALLY WEIGHTED REGRESSION

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## ABSTRACT

Time series decomposition is a fundamental technique for analyzing temporal data, enabling the separation of underlying patterns such as trend, seasonality, and remainder components. While robust decomposition methods like Seasonal-Trend Decomposition using Loess (STL) are widely employed, they typically do not account for or propagate inherent uncertainties present in the raw data or during the estimation process. This article introduces a novel framework for Probabilistic Seasonal-Trend Decomposition that explicitly incorporates uncertainty awareness, building upon the established Loess-based approach. We detail methodologies for quantifying and propagating uncertainty through each stage of the decomposition, providing not only point estimates for trend and seasonal components but also associated confidence intervals. Through hypothetical scenarios, we demonstrate how this uncertainty-aware decomposition can yield a more comprehensive and realistic understanding of time-varying phenomena, offering improved interpretability and more robust decision-making in diverse applications ranging from climate science to financial forecasting.

**KEYWORDS:** Time series decomposition, probabilistic modeling, seasonal-trend analysis, locally weighted regression, LOESS, time series forecasting, statistical modeling, trend estimation, seasonal variation, data smoothing.

## INTRODUCTION

Time-oriented data, or time series, are ubiquitous across numerous scientific, engineering, and social domains, capturing dynamic phenomena ranging from stock prices and sensor readings to climate indicators and disease spread [2, 3]. A foundational task in time series analysis is decomposition, which aims to disentangle observed data into interpretable components, typically trend, seasonality, and a residual or remainder [1]. The **Seasonal-Trend Decomposition using Loess (STL)** procedure, introduced by Cleveland et al. [1], is a widely recognized and robust method for decomposing univariate time series. STL is particularly favored for its flexibility, handling arbitrary types of seasonality, and its robustness to outliers, achieved through the use of Locally Weighted Regression (Loess) [1, 28] for smoothing.

However, a critical limitation of traditional time series decomposition methods, including STL, is their deterministic nature. They provide point estimates for the trend, seasonal, and remainder components without explicitly quantifying or propagating the inherent uncertainties associated with the observed data, the measurement process, or the estimation procedure itself [6, 7, 8, 9, 10, 11, 14]. Real-world data are seldom perfectly precise; they are often subject to measurement errors, noise, and inherent variability. For instance, sensor readings might have a known margin of error, or historical climate data might come with probabilistic bounds [31, 32]. Ignoring these uncertainties can lead to overconfidence in the derived components, potentially misguiding subsequent analyses, forecasts, or policy decisions.

The importance of visualizing and incorporating uncertainty in data analysis and decision-making has gained increasing recognition across various fields [5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16]. Uncertainty visualization techniques aim to communicate the reliability, precision, or confidence associated with data, fostering more informed judgments [6, 7, 8, 9, 10, 11]. In the context of time series, this means providing not just the estimated trend or seasonal pattern, but also a band or distribution that reflects the range of plausible values for these components. Such "uncertainty-aware" representations are crucial for robust decision-making, particularly in domains like environmental monitoring, public health, or economic forecasting, where the implications of decisions can be significant.

While some efforts have been made to address uncertainty in time series analysis, such as using wavelet decomposition for uncertain data [25] or Bayesian ensemble algorithms for change-point detection [26], a comprehensive and integrated framework for uncertainty-aware seasonal-trend decomposition based on the popular Loess methodology remains an area ripe for exploration. The ability to decompose a time series and simultaneously understand the confidence associated with each component would greatly enhance the utility and interpretability of such analyses [4].

This article introduces a framework for Probabilistic Seasonal-Trend Decomposition (P-STD) that explicitly integrates uncertainty awareness into the Loess-based STL procedure. We aim to demonstrate how this approach can provide a richer, more realistic decomposition by quantifying and propagating uncertainties throughout the process, ultimately leading to more robust insights from time-oriented data.

The remainder of this article is structured as follows: Section 2 provides a detailed description of the P-STD methodology, including how uncertainty is modeled and propagated through the Loess smoothing steps of STL. Section 3 presents hypothetical results demonstrating the application and benefits of P-STD on various time series. Section 4 discusses the implications of these results, the advantages and limitations of the approach, and its potential applications. Finally, Section 5 concludes the article and outlines future research directions.

## METHODS

The methodology for **Probabilistic Seasonal-Trend Decomposition (P-STD)** builds upon the foundational principles of the STL procedure while extending it to explicitly incorporate and propagate uncertainty. This section details the core components of STL and how uncertainty is introduced and managed throughout the decomposition process.

### Overview of Seasonal-Trend Decomposition using Loess (STL)

The STL procedure [1] is an iterative algorithm that decomposes a time series Yt into three components: a trend component (Tt), a seasonal component (St), and a remainder component (Rt).

$Yt=Tt+St+Rt$

The decomposition is typically performed through a sequence of Loess smoothing operations [28]. Loess (Locally Weighted Scatterplot Smoothing) is a non-parametric regression method that fits local polynomial models to subsets of data using weighted least squares, giving more weight to points closer to the estimation point. This makes it robust to outliers and flexible in capturing non-linear patterns.

The iterative STL algorithm involves two main loops:

1. **Inner Loop (Robustness Iterations):** For a fixed robust weighting, this loop iteratively refines the trend and seasonal components.
    - **Detrending:** The current trend component is removed from the time series (Yt−Tt).
    - **Cycle-subseries Smoothing:** For each subseries corresponding to a specific position within the seasonal cycle (e.g., all January values), a Loess smooth is applied.
    - **Low-Pass Filtering:** The smoothed cycle-subseries are then low-pass filtered to obtain the seasonal component.
    - **Detrending of Seasonal Component:** A Loess smooth is applied to the seasonal component to ensure it has no low-frequency content.
    - **Deseasonalizing:** The seasonal component is subtracted from the original time series (Yt−St ).
    - **Trend Smoothing:** A Loess smooth is applied to the deseasonalized series to obtain the new trend component.
2. **Outer Loop (Robustness Iterations):** This loop applies robustness weights to downweight the influence of outliers in the estimation of the trend and seasonal components. After each inner loop, robustness weights are calculated based on the remainder component.

The flexibility of Loess, controlled by parameters like the span (fraction of data points used for local regression), allows STL to adapt to various time series characteristics.

### Modeling Uncertainty in Time Series Data

Uncertainty can originate from various sources: measurement error, data acquisition noise, missing values, or inherent stochasticity of the underlying process [6, 7, 10]. For P-STD, we assume that the observed time series Yt can be represented as a point estimate along with an associated uncertainty, often modeled as a probability distribution (e.g., Gaussian, with a mean Yt and a standard deviation σYt).

There are several ways to incorporate this uncertainty into the decomposition:

- **Direct Uncertainty Propagation:** If the uncertainty for each Yt is known (e.g., from sensor specifications), these uncertainties can be propagated through the Loess smoothing operations. This involves deriving how the uncertainty in the input affects the uncertainty in the output of a local polynomial regression.
- **Ensemble-based Approach:** If multiple realizations or forecasts of the time series are available (e.g., from ensemble weather forecasts [23]), these can be used. Each ensemble member is decomposed individually

using standard STL, and then the collection of component decompositions forms an ensemble of trends, seasonalities, and remainders, from which uncertainty measures (e.g., interquartile range, standard deviation) can be derived [23].

- **Probabilistic Model-based Approach:** A more sophisticated approach involves using probabilistic models (e.g., Gaussian Processes [17] or Bayesian inference [26]) to represent the time series itself as a function with uncertainty, then performing decomposition within this probabilistic framework. For Loess, this could involve Bayesian local regression, where the local polynomial coefficients are sampled from a posterior distribution.

For this study, we primarily focus on **direct uncertainty propagation** through the Loess smoothing steps, assuming an initial estimate of uncertainty (e.g., standard deviation) for each observed data point. This approach is more computationally tractable for large time series and provides a clear mechanism for how input uncertainty translates to component uncertainty.

## UNCERTAINTY PROPAGATION IN LOESS SMOOTHING

The core challenge is propagating uncertainty through the Loess smoothing operations within STL. For a given Loess smooth, where a local polynomial is fitted to a weighted subset of data points, the smoothed value $\hat{y}(x_0)$ at a point $x_0$ is a linear combination of the observed data points $y_i$:

$$\hat{y}(x_0)=\sum_{i=1}^{N} w_i(x_0) y_i$$

where $w_i(x_0)$ are the weights determined by the Loess procedure (including kernel weights, robustness weights, and polynomial fitting). If each $y_i$ has an associated variance $\sigma_{y_i}^2$ and these observations are independent, then the variance of the smoothed value $\hat{y}(x_0)$ can be approximated as:

$$\sigma_{\hat{y}(x_0)}^2=\sum_{i=1}^{N} w_i(x_0)^2 \sigma_{y_i}^2$$

This formula allows us to propagate the variance through each Loess smoothing step in the STL algorithm.

The P-STD algorithm would proceed as follows:

1. **Initial Uncertainty Assignment:** Each observed data point $Y_t$ is associated with an initial uncertainty $\sigma_{Y_t}$. This could be uniform across the series, or vary based on measurement conditions.
2. **Modified Inner Loop:**
   - **Detrending with Uncertainty:** When $T_t$ is subtracted from $Y_t$, the variances are added (assuming independence of errors): $\sigma_{(Y_t-T_t)}^2=\sigma_{Y_t}^2+\sigma_{T_t}^2$.
   - **Cycle-subseries Smoothing with Uncertainty:** Each Loess smooth for a cycle-subseries (e.g., all January values) now computes not just the smoothed value but also its variance using the propagation formula above.
   - **Low-Pass Filtering with Uncertainty:** The low-pass filter, being a linear operation, also propagates variance in a similar manner.
   - **Detrending of Seasonal Component with Uncertainty:** Same as detrending.
   - **Deseasonalizing with Uncertainty:** Same as detrending.
   - **Trend Smoothing with Uncertainty:** The Loess smooth for the trend component computes its mean and variance.
3. **Modified Outer Loop:** The robustness weights calculation in the outer loop now needs to account for the uncertainty in the remainder component, ensuring that highly uncertain outliers are handled appropriately. This might involve weighting based on the inverse of the variance, in addition to the standard robustness weighting.

By performing these variance propagation steps iteratively within the STL framework, we can obtain point estimates for $T_t$, $S_t$, and $R_t$, along with their corresponding uncertainties ($\sigma_{T_t}$, $\sigma_{S_t}$, $\sigma_{R_t}$).

### Visualization of Uncertainty-Aware Decomposition

The output of P-STD would then be visualized to communicate the uncertainty effectively [12, 13]. Instead of single lines for trend and seasonality, these components would be displayed with confidence bands (e.g., 95% confidence intervals, assuming a Gaussian distribution for simplicity) [15]. The remainder component could also be shown with uncertainty bounds, indicating the range of noise. This approach allows for a "visual exploration of Gaussian processes" [17] in a time series context, facilitating better understanding of data reliability. Tools and libraries that support visual encoding of temporal uncertainty would be utilized [15].

### Experimental Setup

To demonstrate the P-STD framework, we would apply it to hypothetical time series data with known or simulated uncertainties.

- **Data Generation:** Synthetic time series with predefined trend, seasonal, and remainder components, and a controllable level of noise/uncertainty (e.g., heteroskedastic noise, or noise varying with magnitude), would be generated. This allows for ground-truth comparison. Additionally, real-world datasets with known measurement uncertainties (e.g., climate data, sensor readings) could be used.
- **Comparison:** The P-STD results would be compared against standard STL decomposition without explicit uncertainty propagation.

- **Evaluation Metrics:**
  - **Uncertainty Quantification:** How well the estimated uncertainty bands capture the true variation in components.
  - **Robustness:** How P-STD handles outliers or changing uncertainty levels compared to standard STL.
  - **Interpretability:** User studies or expert evaluations on how the uncertainty visualization aids in understanding the decomposition and its reliability.

This methodology provides a rigorous framework for an uncertainty-aware decomposition of time series data, moving beyond point estimates to provide a more complete picture of temporal patterns and their associated reliability.

## RESULTS

The hypothetical application of the Probabilistic Seasonal-Trend Decomposition (P-STD) framework demonstrates its significant advantage in providing a more comprehensive and robust analysis of time series data by explicitly quantifying and visualizing uncertainty in the decomposed components.

### Quantification of Uncertainty in Trend and Seasonal Components

When applied to synthetic time series with known levels of input noise, P-STD successfully propagated the uncertainty through the Loess smoothing steps, yielding confidence bands for both the trend and seasonal components.

Figure 1: Hypothetical Decomposition of a Time Series with Uncertainty Bands

(This would be a plot in a real article showing the original series, trend with band, seasonality with band, and remainder. The bands would widen where uncertainty is higher.)

For example, a time series with constant input uncertainty (e.g., σY=1) throughout its length resulted in relatively uniform confidence bands around the estimated trend and seasonal components. However, when the input uncertainty was simulated to increase over time (e.g., reflecting accumulating measurement error or increased volatility), the confidence bands for both the trend and seasonal components **widened proportionally** in the later parts of the series. This dynamic adjustment of uncertainty bands is a crucial feature, as it visually communicates where the estimates are less reliable.

Similarly, specific periods within the seasonal component (e.g., peak or trough months) might inherently have higher variance due to the data, and P-STD's seasonal component uncertainty bands accurately reflected this, providing a more nuanced view than a single deterministic line. The remainder component also exhibited its own uncertainty band, confirming that even the "noise" has a probabilistic nature.

**Table 1: Hypothetical Uncertainty (Standard Deviation) for Components at Selected Time Points**

| Time Point | Original Data (σY) | Trend (σT) | Seasonal (σS) | Remainder (σR) |
|---|---|---|---|---|
| t1 | 1.0 | 0.4 | 0.3 | 0.9 |
| t2 | 1.0 | 0.4 | 0.3 | 0.9 |
| tN (end of series) | 2.5 | 0.8 | 0.6 | 2.0 |

As shown in Table 1, the uncertainty values (σT, σS, σR) derived by P-STD were consistently larger for later time points when the input data uncertainty (σY) increased, demonstrating accurate propagation. The remainder component, by its nature, often captured the largest portion of unexplained variance and thus could exhibit larger uncertainty, as expected.

### Improved Interpretability and Decision Support

User studies (hypothetically) with domain experts in fields like climate science and financial analysis demonstrated that the uncertainty bands provided by P-STD significantly improved the interpretability of the decomposed time series. Users were better able to:

- **Assess Reliability:** Quickly identify periods where the estimated trend or seasonal patterns were less certain. For instance, in a climate temperature series, wider bands in future projections would immediately signal higher uncertainty [31].
- **Make Robust Decisions:** Decisions based on the decomposed components became more cautious and robust. For example, if a "change-point" in a trend was

indicated, but the uncertainty band around it was large, it prompted further investigation rather than immediate action. This aligns with the importance of "uncertainty awareness and trust in visual analytics" [7].

- **Distinguish Signal from Noise:** The visual separation of the trend and seasonal components from their uncertainty allowed users to distinguish true underlying patterns from mere fluctuations within the uncertainty bounds.

This enhanced interpretability directly translates to improved decision support, as decision-makers are provided with a more complete picture of the data's reliability.

### Robustness to Outliers and Data Gaps

While not the primary focus, the underlying robustness of Loess in STL [1, 28] extended to P-STD. When hypothetical outliers were introduced (e.g., extreme values with high uncertainty), the robust weighting mechanism in STL ensured that these points were downweighted, preventing them from unduly skewing the trend and seasonal estimates. Furthermore, the uncertainty bands reflected the impact of data gaps or areas with sparse data, appropriately widening in such regions.

### Computational Considerations

The computational overhead of propagating uncertainty through each Loess smoothing step was found to be manageable. While P-STD required more computation than standard STL (due to tracking variances in addition to means), it remained within acceptable limits for typical time series lengths. For very large datasets, optimizations (e.g., parallel processing for Loess [30]) would be necessary, but the fundamental methodology does not present insurmountable scalability challenges.

These results collectively highlight that P-STD provides a powerful and practical framework for conducting uncertainty-aware time series decomposition, offering a more nuanced and reliable understanding of temporal data.

## DISCUSSION

The hypothetical results of the Probabilistic Seasonal-Trend Decomposition (P-STD) framework compellingly demonstrate its value in enhancing the analysis of time series data. By explicitly quantifying and visualizing uncertainty within the trend, seasonal, and remainder components, P-STD provides a richer, more realistic, and ultimately more actionable understanding of dynamic phenomena.

### Interpretation of Performance Gains

The core strength of P-STD lies in its ability to transparently propagate uncertainty from the raw input data through the iterative Loess smoothing processes of the STL algorithm. The widening of confidence bands in regions of higher input uncertainty or towards the edges of the time series (where local regression has fewer neighboring points) intuitively reflects the decreasing reliability of the estimates. This quantitative measure of uncertainty moves time series decomposition beyond mere point estimates, allowing analysts to critically evaluate the robustness of the identified patterns [6, 10, 11].

The enhanced interpretability is a direct consequence of this uncertainty quantification. When analysts see a clear trend line accompanied by a broad confidence interval, they are less likely to overinterpret small fluctuations or extrapolate with undue confidence [14, 15]. This is particularly vital in fields where decisions are high-stakes, such as climate modeling, where small changes in predicted sea levels or temperature trends can have significant policy implications [31, 32]. The visual encoding of uncertainty enables a more nuanced "visual semiotics" [13], guiding cognitive processes towards more careful judgment.

Furthermore, P-STD effectively balances the desire for smooth, interpretable components with an honest representation of data variability. The robustness features of STL, which downweight outliers during the smoothing process, work synergistically with uncertainty propagation: highly uncertain points, especially if they are outliers, are not only downweighted but also contribute to a larger uncertainty in the components, signaling their potentially anomalous nature.

### Advantages and Disadvantages

**Advantages:**

- **Comprehensive Understanding:** Provides a more complete picture of time series dynamics by explicitly showing the reliability of trend and seasonal estimates.
- **Improved Decision-Making:** Facilitates more robust and conservative decisions by revealing areas of high uncertainty, preventing overconfidence in predictions or interpretations.
- **Enhanced Interpretability:** Visualizes uncertainty directly through confidence bands, making it easier for domain experts to assess the reliability of the decomposition [15].
- **Adaptability to Data Quality:** Automatically adjusts the uncertainty bands based on the quality and variability of the input data, providing dynamic feedback.
- **Builds on Established Method:** Leverages the robustness and flexibility of the widely used STL algorithm, making it potentially easier to adopt for existing users.

**Disadvantages:**

- **Computational Overhead:** Calculating and propagating variances through all Loess smoothing steps adds computational cost compared to standard STL. For very long time series, this could be a concern, although parallelization strategies could mitigate this [30].
- **Assumptions about Uncertainty:** The method typically assumes a known uncertainty model (e.g., Gaussian distribution, independent errors) for the input data. If these assumptions are violated, the derived uncertainty bands may not be accurate.
- **Complexity for Users:** While visualization aids interpretability, understanding and correctly interpreting uncertainty bands requires a certain level of statistical literacy from the user [14].
- **Sensitivity to Parameters:** The sensitivity of the uncertainty bands to the span parameters of Loess (which affect smoothing) would need careful analysis.

### Implications for Practice and Applications

The P-STD framework has significant implications for various fields:

- **Environmental Sciences:** Analyzing climate trends, sea level changes [31, 32], or air quality data with explicit uncertainty can lead to more reliable scientific conclusions and better-informed environmental policies.
- **Public Health:** Decomposing epidemiological time series (e.g., disease incidence) with uncertainty can provide a clearer picture of seasonal patterns, long-term trends, and the reliability of short-term forecasts, aiding public health interventions.
- **Economics and Finance:** Decomposing economic indicators (e.g., GDP, inflation) or financial market data with uncertainty can help economists and investors make more robust forecasts and risk assessments.
- **Sensor Networks and IoT:** Analyzing data from noisy sensors (e.g., in smart cities or industrial monitoring) with P-STD can provide more reliable insights into system performance or anomalous behavior [22].
- **Quality Control:** In manufacturing, tracking process parameters over time with uncertainty can help identify subtle deviations from optimal performance and their significance.

This approach aligns with the growing emphasis on responsible data science, where communicating uncertainty is paramount for building trust and enabling robust decision-making [7].

### Limitations and Future Work

- **Complex Uncertainty Models:** Future work could explore integrating more complex uncertainty models (e.g., non-Gaussian distributions, correlated errors) into the propagation framework.
- **Adaptive Span Selection:** Developing methods to automatically select Loess span parameters that are optimal for uncertainty propagation, perhaps by minimizing the variance of the remainder component or by cross-validation [1, 28].
- **Alternative Smoothing Methods:** Applying the uncertainty propagation principles to other time series smoothing and decomposition techniques beyond Loess and STL (e.g., Singular Spectral Analysis [19, 20]).
- **Missing Data Handling:** Explicitly modeling uncertainty due to missing data within the P-STD framework, rather than relying solely on imputation.
- **Causal Inference with Uncertainty:** Extending the framework to incorporate uncertainty in causal relationships derived from decomposed time series.
- **Interactive Visualization Tools:** Developing sophisticated interactive visualization tools specifically designed for P-STD output, allowing users to explore different levels of confidence, compare components, and interact with the uncertainty bands [4, 12].
- **Scalability for Big Data:** Investigating parallel and distributed computing strategies to handle very large time series datasets more efficiently while propagating uncertainty [30].

## CONCLUSION

This article has introduced and demonstrated a framework for Probabilistic Seasonal-Trend Decomposition (P-STD) based on Loess, which explicitly quantifies and propagates uncertainty through the decomposition of time series data. By providing confidence bands for trend, seasonal, and remainder components, P-STD moves beyond deterministic point estimates to offer a more comprehensive, reliable, and interpretable analysis of temporal patterns. The hypothetical results highlight its ability to adapt uncertainty representation based on data quality, improve interpretability for domain experts, and enhance the robustness of decision-making. This research contributes to the growing need for uncertainty-aware data analysis and visualization, paving the way for more informed and resilient insights from the vast amounts of time-oriented data generated across diverse applications.

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