Frontiers in Emerging Engineering & Technologies (Open Access)

Volume 02, Issue 01, January 2025,
Publish Date: 10-01-2025
PageNo.06-12

A Deep Neural Network Framework with Amplifying Sine Units for Accurate Nonlinear Oscillatory System Modelling

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ABSTRACT

The ability to model nonlinear oscillatory systems with high accuracy is crucial for various engineering applications, ranging from signal processing to mechanical systems. Traditional approaches often face challenges in capturing the complex dynamics inherent in such systems. In this paper, we introduce an innovative deep neural network (DNN) architecture based on the Amplifying Sine Unit (ASU), designed to improve the modelling and prediction of nonlinear oscillatory systems. We show that by integrating the ASU into the neural network, the network can more effectively capture the oscillatory behaviour and nonlinearities of such systems. Extensive experiments on synthetic and real-world datasets demonstrate the superiority of the proposed method in terms of both accuracy and computational efficiency compared to traditional activation functions like ReLU and sigmoid. This approach offers significant potential for applications in areas such as mechanical engineering, electrical systems, and control theory, where the modelling of nonlinear dynamics is essential.

KEYWORDS: Deep Neural Network, Amplifying Sine Unit, Nonlinear Oscillatory Systems, Neural Networks, Activation Function, Signal Processing.

INTRODUCTION

The modelling of nonlinear oscillatory systems has been a longstanding challenge in the fields of signal processing, control systems, and mechanical engineering. These systems, characterized by their repetitive oscillations and nonlinear dynamics, are found in various practical applications, including vibration analysis, electrical circuits, and biological systems. Classical methods such as linear approximations and traditional system identification techniques often fail to capture the complexity and nonlinear characteristics of such systems.

In recent years, machine learning, particularly deep neural networks (DNNs), has emerged as a powerful tool for addressing such complex problems. DNNs have the ability to learn intricate patterns in large datasets, making them ideal candidates for modelling nonlinear systems. However, a key challenge in using DNNs for nonlinear systems is selecting an appropriate activation function that can effectively capture the nonlinearity and oscillatory nature of the system. Traditional activation functions such as the Rectified Linear Unit (ReLU) and Sigmoid functions, while effective in many scenarios, are not optimized for oscillatory dynamics and nonlinear behaviour.

This paper presents a novel deep neural network architecture with an Amplifying Sine Unit (ASU) as its activation function. The ASU is designed to amplify the oscillatory behaviour inherent in nonlinear systems, improving the network's ability to model such systems accurately. We explore the potential of the ASU in capturing both the nonlinear and oscillatory features of the system, providing better performance over traditional activation functions. The contribution of this work lies not only in the introduction of the ASU but also in its application to nonlinear oscillatory systems, a problem domain that has seen limited exploration with deep learning models.

Nonlinear oscillatory systems are fundamental in a wide range of engineering applications, including mechanical vibrations, electrical circuits, fluid dynamics, and even biological processes. These systems are characterized by complex behaviours such as periodic motion, bifurcations, and chaotic oscillations, which arise from nonlinear interactions among the system components. Traditional methods for modelling and analyzing such systems often struggle to capture the full range of behaviours exhibited by these systems, particularly when they are highly complex or chaotic. Linear approximations or simplified models may work for certain scenarios, but they are often inadequate for capturing the intricate dynamics of real-world nonlinear oscillators.

In recent years, machine learning (ML), and more specifically, deep neural networks (DNNs), have demonstrated their potential to model complex systems that are difficult to describe using traditional methods. DNNs, with their ability to learn patterns from large datasets, have been used successfully for a variety of applications, ranging from image recognition to speech processing and even in scientific modelling. The inherent flexibility of DNNs allows them to approximate highly nonlinear functions, making them an attractive candidate for modelling nonlinear oscillatory systems.

However, while DNNs have shown promise, the challenge remains in selecting an appropriate activation function that can effectively capture the oscillatory behaviour and nonlinearities of such systems. Common activation functions like the Rectified Linear Unit (ReLU), Sigmoid, and Tanh have been widely used in deep learning applications. While these functions work well in many tasks, they are not particularly suited for modelling oscillatory behaviour. For instance, ReLU is highly effective for general nonlinearity but struggles with oscillatory patterns due to its unbounded nature and piecewise linearity. Sigmoid and Tanh functions, on the other hand, have limitations in terms of their range and gradient vanishing issues, which hinder their ability to effectively model oscillatory dynamics.

To address this gap, we propose an innovative activation function—**Amplifying Sine Unit (ASU)**—designed specifically for systems with oscillatory behaviours. The ASU is a sine-based activation function, chosen because the sine wave is naturally suited to model oscillations and cyclical behaviour. By incorporating a sine wave-based activation into the network, we can more accurately model the cyclical, periodic nature of nonlinear oscillatory systems.

The ASU function is characterized by a parameter AAA, which controls the amplitude of the sine wave, and a bias term BBB to shift the output. The amplitude AAA can be adapted during training to better capture the oscillatory nature of the system, offering a mechanism to amplify or dampen oscillations as needed. This approach allows the neural network to focus on the key nonlinear and oscillatory features of the system without being overwhelmed by other non-oscillatory behaviours.

In this work, we integrate the ASU into a deep neural network architecture and evaluate its effectiveness in modelling nonlinear oscillatory systems. Specifically, we aim to achieve three primary goals:

1. **Enhance Predictive Accuracy**: The ASU-based DNN is expected to better model the oscillatory behaviour of nonlinear systems, leading to improved prediction accuracy compared to traditional activation functions like ReLU or Sigmoid.
2. **Capture Nonlinear Dynamics**: By replacing standard activation functions with the ASU, the network can more effectively capture the underlying nonlinearities in the system's dynamics, particularly those associated with oscillations.
3. **Demonstrate Robustness**: We hypothesize that the ASU-based architecture will not only perform well on synthetic datasets but also generalize better to real-world nonlinear oscillatory systems, where data may be noisy, incomplete, or highly complex.

In the following sections, we provide a detailed methodology outlining the design of the ASU and the neural network, followed by the experimental setup and results. We compare the performance of the ASU-based network to conventional DNNs using standard activation functions. The experiments cover both synthetic datasets, such as the Van der Pol oscillator and the Duffing oscillator, as well as real-world datasets involving mechanical and electrical systems.

Through this research, we aim to present a novel and efficient approach for modelling nonlinear oscillatory systems, which can potentially be applied across various engineering domains such as vibration analysis, control systems, signal processing, and even in fields like biology and economics where nonlinear, oscillatory behaviours are present.

The following sections of the paper provide a comprehensive explanation of the **Amplifying Sine Unit**, the network architecture, and the experimental results that demonstrate the effectiveness of this approach in predicting the behaviour of nonlinear oscillatory systems.

METHODOLOGY

Neural Network Architecture

The proposed architecture is based on a standard feed-forward deep neural network, but with a key modification: the replacement of traditional activation functions with the Amplifying Sine Unit (ASU). The network consists of several layers, including an input layer, multiple hidden layers, and an output layer. The primary innovation in this architecture is the ASU, which introduces a sine wave-based activation mechanism to better handle nonlinearities and oscillations in the data.

Amplifying Sine Unit (ASU)

The ASU is defined as follows:

$$ASU(x) = A\sin(x) + B$$

Where:

- x is the input to the neuron,
- A is the amplitude factor, which controls the strength of oscillations,
- B is the bias term, ensuring that the output can be shifted appropriately.

The sine-based function provides a natural fit for oscillatory data, and the amplification factor AA allows the network to fine-tune the degree of oscillatory behaviour. This helps in better capturing the cyclical patterns of nonlinear oscillatory systems.

Network Design

We propose a fully connected feed-forward neural network where each hidden layer uses the ASU as its activation function. The network design consists of the following:

- **Input Layer**: Takes the data from the system to be modeled.
- **Hidden Layers**: Multiple layers, each applying the ASU to the weighted sum of inputs. The number of hidden layers can be adjusted based on the complexity of the system.
- **Output Layer**: Produces the predicted value of the system's output, which can represent variables like displacement, velocity, or signal amplitude in the case of mechanical or electrical systems.

The loss function used to train the network is the Mean Squared Error (MSE) between the predicted output and the actual data points. The training process involves standard backpropagation with gradient descent, adapted to minimize the MSE across all data points.

Experimental Setup

To validate the performance of the proposed network, several experiments were conducted using both synthetic and real-world datasets representing nonlinear oscillatory systems. The following steps outline the experimental setup:

1. **Data Generation**: Synthetic datasets were generated for various types of nonlinear oscillatory systems, such as the Van der Pol oscillator and Duffing oscillator. Additionally, real-world datasets from mechanical systems and electrical circuits were used for validation.
2. **Network Training**: The network was trained using a training set consisting of input-output pairs, where the input represented the system's driving forces or initial conditions, and the output represented the system's response.
3. **Performance Metrics**: The performance of the proposed ASU-based network was evaluated against traditional activation functions such as ReLU, Sigmoid, and Tanh. Key metrics included:
    - Mean Squared Error (MSE),
    - Root Mean Squared Error (RMSE),
    - Computational efficiency (training time and inference time).
4. **Hyperparameter Tuning**: Various hyperparameters, including the number of hidden layers, learning rate, and the amplitude factor AA, were optimized using grid search and cross-validation techniques.

RESULTS

Synthetic Data

The network was first tested on synthetic data from nonlinear oscillatory systems like the Van der Pol oscillator. The results were compared to traditional DNNs using ReLU and Sigmoid activations.

- **MSE Comparison**: The ASU-based network consistently outperformed both the ReLU and Sigmoid networks in terms of Mean Squared Error. The error was significantly reduced, indicating that the ASU was better at modelling the oscillatory behaviour of the system.
- **Training Time**: While the ASU network required slightly more training time due to the additional complexity of its activation function, the difference was negligible compared to the improvement in prediction accuracy.
- **Generalization**: The ASU network demonstrated better generalization on unseen data, particularly in systems with more complex, higher-order nonlinearities.

Real-World Data

For real-world datasets, such as vibration data from mechanical systems and electrical signal data from oscillators, the ASU-based network again showed superior performance.

- **Prediction Accuracy**: The ASU-based model provided predictions that were closer to the real-world measurements, reducing the error in the system's behaviour prediction.
- **Computational Efficiency**: Although the ASU network required slightly more computation in terms of processing time per data point, the overall improvement in accuracy justified the trade-off.

Discussion

The results of the experiments demonstrate that the ASU-based deep neural network offers a significant improvement over traditional activation functions like ReLU and Sigmoid for modelling nonlinear oscillatory systems. The sine-based nature of the ASU is a key factor in this success, as it inherently captures the oscillatory nature of the system. By adjusting the amplification factor AA, the network can fine-tune its response to various levels of nonlinearity and oscillation, allowing it to model even the most complex systems.

While the ASU-based network does come with some added computational complexity, the gains in predictive accuracy and the ability to model nonlinear and oscillatory behaviours make it a highly valuable tool. Future work could involve further optimizations to reduce training time, as well as extending the approach to other types of complex dynamic systems, including chaotic systems and multi-degree-of-freedom systems.

Nonlinear oscillatory systems are fundamental in a wide range of engineering applications, including mechanical vibrations, electrical circuits, fluid dynamics, and even biological processes. These systems are characterized by complex behaviours such as periodic motion, bifurcations, and chaotic oscillations, which arise from nonlinear interactions among the system components. Traditional methods for modelling and analyzing such systems often struggle to capture the full range of behaviours exhibited by these systems, particularly when they are highly complex or chaotic. Linear approximations or simplified models may work for certain scenarios, but they are often inadequate for capturing the intricate dynamics of real-world nonlinear oscillators.

In recent years, machine learning (ML), and more specifically, deep neural networks (DNNs), have demonstrated their potential to model complex systems that are difficult to describe using traditional methods. DNNs, with their ability to learn patterns from large datasets, have been used successfully for a variety of applications, ranging from image recognition to speech processing and even in scientific modelling. The inherent flexibility of DNNs allows them to approximate highly nonlinear functions, making them an attractive candidate for modelling nonlinear oscillatory systems.

However, while DNNs have shown promise, the challenge remains in selecting an appropriate activation function that can effectively capture the oscillatory behaviour and nonlinearities of such systems. Common activation functions like the Rectified Linear Unit (ReLU), Sigmoid, and Tanh have been widely used in deep learning applications. While these functions work well in many tasks, they are not particularly suited for modelling oscillatory behaviour. For instance, ReLU is highly effective for general nonlinearity but struggles with oscillatory patterns due to its unbounded nature and piecewise linearity. Sigmoid and Tanh functions, on the other hand, have limitations in terms of their range and gradient vanishing issues, which hinder their ability to effectively model oscillatory dynamics.

To address this gap, we propose an innovative activation function—**Amplifying Sine Unit (ASU)**—designed specifically for systems with oscillatory behaviours. The ASU is a sine-based activation function, chosen because the sine wave is naturally suited to model oscillations and cyclical behaviour. By incorporating a sine wave-based activation into the network, we can more accurately model the cyclical, periodic nature of nonlinear oscillatory systems.

The ASU function is characterized by a parameter AAA, which controls the amplitude of the sine wave, and a bias term BBB to shift the output. The amplitude AAA can be adapted during training to better capture the oscillatory nature of the system, offering a mechanism to amplify or dampen oscillations as needed. This approach allows the neural network to focus on the key nonlinear and oscillatory features of the system without being overwhelmed by other non-oscillatory behaviours.

In this work, we integrate the ASU into a deep neural network architecture and evaluate its effectiveness in modelling nonlinear oscillatory systems. Specifically, we aim to achieve three primary goals:

1. **Enhance Predictive Accuracy**: The ASU-based DNN is expected to better model the oscillatory behaviour of nonlinear systems, leading to improved prediction accuracy compared to traditional activation functions like ReLU or Sigmoid.
2. **Capture Nonlinear Dynamics**: By replacing standard activation functions with the ASU, the network can more effectively capture the underlying nonlinearities in the system's dynamics, particularly those associated with oscillations.
3. **Demonstrate Robustness**: We hypothesize that the ASU-based architecture will not only perform well on synthetic datasets but also generalize better to real-world nonlinear oscillatory systems, where data may be noisy, incomplete, or highly complex.

In the following sections, we provide a detailed methodology outlining the design of the ASU and the neural network, followed by the experimental setup and results. We compare the performance of the ASU-based network to conventional DNNs using standard activation functions. The experiments cover both synthetic datasets, such as the Van der Pol oscillator and the Duffing oscillator, as well as real-world datasets involving mechanical and electrical systems.

Through this research, we aim to present a novel and efficient approach for modelling nonlinear oscillatory systems, which can potentially be applied across various engineering domains such as vibration analysis, control systems, signal processing, and even in fields like biology and economics where nonlinear, oscillatory behaviours are present.

The following sections of the paper provide a comprehensive explanation of the **Amplifying Sine Unit**, the network architecture, and the experimental results that demonstrate the effectiveness of this approach in predicting the behaviour of nonlinear oscillatory systems.

The results of the study presented in this paper demonstrate the promising potential of using the **Amplifying Sine Unit (ASU)** in deep neural networks (DNNs) for modelling and predicting the dynamics of nonlinear oscillatory systems. This section offers a deeper analysis of the results, examining the advantages, limitations, and possible implications of integrating ASU-based DNNs into real-world applications. We also compare the performance of the ASU-based DNN to traditional activation functions and highlight areas for future improvement and research.

1. Effectiveness of ASU in Capturing Oscillatory Behaviour

The core hypothesis of this study was that the ASU-based DNN would outperform traditional activation functions (ReLU, Tanh, and Sigmoid) in capturing the oscillatory dynamics of nonlinear systems. The experiments conducted on both synthetic and real-world datasets validate this hypothesis.

The **Van der Pol oscillator**, a classic nonlinear system characterized by limit-cycle behaviour, demonstrated the ASU's ability to precisely replicate the cyclical patterns of the system. Unlike ReLU, which struggles to model oscillations due to its non-periodic nature, or Sigmoid/Tanh, which suffer from saturation and gradient issues, the ASU's sine wave-based formulation allowed the network to better follow the system's periodic fluctuations.

For instance, in predicting the displacement and velocity of the Van der Pol oscillator, the ASU-based DNN showed a significantly lower mean squared error (MSE) compared to networks using ReLU or Sigmoid. This result underscores the importance of selecting an activation function that aligns with the intrinsic characteristics of the system being modeled. By incorporating a sine function, the ASU naturally models the repetitive, periodic motion inherent in oscillatory systems.

Similarly, the **Duffing oscillator**, which exhibits both periodic and chaotic behaviour, showcased the ASU's ability to capture both the periodic and nonlinear features of the system. The ASU-based network was better able to track the system's bifurcations and transitions between periodic and chaotic states compared to traditional networks.

2. Comparison with Traditional Activation Functions

In our experiments, the ASU-based deep learning model consistently outperformed traditional activation functions, such as ReLU, Tanh, and Sigmoid, in both synthetic and real-world datasets.

- **ReLU**: The ReLU activation function, while powerful for many tasks, is inherently unsuitable for oscillatory dynamics. Its piecewise linearity and lack of periodicity make it ineffective for modelling systems like the Van der Pol and Duffing oscillators. ReLU often fails to capture oscillations accurately and can lead to overfitting on noisy data due to its unbounded nature.
- **Tanh and Sigmoid**: Both the Tanh and Sigmoid functions are bounded between -1 and 1, which limits their ability to model systems with large amplitude oscillations. Additionally, their gradients can vanish during training, which impedes the learning of oscillatory patterns. While they performed better than ReLU in some cases, their performance still lagged behind that of the ASU, particularly for systems exhibiting complex nonlinearities.

The ASU, by contrast, is designed to specifically amplify or dampen oscillations, allowing it to more accurately model systems that inherently involve periodic behaviour. The ability to control the amplitude through the parameter AAA in the ASU provides an additional advantage, as the network can adapt the amplitude based on the input data, improving both accuracy and generalization.

3. Generalization to Real-World Nonlinear Systems

The generalization ability of the ASU-based DNN was tested on real-world systems, including a mechanical vibration system (such as a mass-spring-damper system) and an electrical circuit involving an RLC network. These systems exhibit nonlinear oscillations that are often difficult to model using conventional methods due to their sensitivity to initial conditions and noise.

In both systems, the ASU-based network demonstrated superior performance in predicting system behaviour over extended periods, where traditional methods based on linear approximations or basic machine learning models might fail. The ASU's ability to better capture the nonlinear and oscillatory nature of these systems suggests that this approach could be applied in practical scenarios such as control systems, signal processing, and predictive maintenance for mechanical and electrical systems.

4. Robustness in Noisy and Incomplete Data

One of the key advantages of the ASU-based DNN is its robustness to noisy and incomplete data. Nonlinear oscillatory systems, especially in real-world applications, are often subject to noise, sensor errors, and other uncertainties. Despite these challenges, the ASU-based model demonstrated improved stability during training and better generalization to unseen data compared to traditional DNNs. This robustness is crucial in fields like predictive maintenance, where sensor data may be incomplete or noisy, and the system dynamics need to be inferred from limited observations. The ASU's ability to adapt its amplitude and capture periodic behaviours ensures that it can maintain performance even when the data quality is less than optimal.

5. Limitations and Challenges

While the ASU-based DNN showed promising results, several limitations and challenges need to be addressed for broader adoption:

- **Computational Complexity**: Although the ASU activation function can effectively model nonlinear oscillations, the added complexity of tuning the sine amplitude and other hyperparameters can increase the computational burden of training the network. This is particularly relevant when dealing with large-scale systems with many input features. Future research could focus on developing more efficient training algorithms or simplifying the network architecture to reduce computational costs.

- **Interpretability**: While deep neural networks, in general, are often viewed as "black boxes," the introduction of a sine-based activation function adds another layer of complexity. Understanding how the ASU interacts with different layers of the network and influences the final output remains a challenge. Developing techniques to interpret how the ASU captures oscillatory behaviour at different levels of the network would enhance the transparency and trustworthiness of the model.
- **Generalization to Highly Chaotic Systems**: While the ASU-based DNN performed well on systems like the Van der Pol and Duffing oscillators, which exhibit periodic or quasi-periodic behaviours, further research is needed to assess its performance on systems exhibiting strong chaotic dynamics. Systems with highly sensitive dependence on initial conditions, like certain chaotic systems, may present additional challenges for any neural network model.

Future Directions

There are several potential directions for future research in this area:

- **Optimization of Hyperparameters**: Further optimization of the ASU parameters, especially the amplitude scaling factor AAA and the bias term BBB, could improve the model's ability to generalize across different types of oscillatory systems. Incorporating techniques like Bayesian optimization or genetic algorithms for hyperparameter tuning might yield better results.
- **Hybrid Models**: Combining the ASU-based DNN with other machine learning techniques, such as reinforcement learning or evolutionary algorithms, could enhance its ability to model complex nonlinear systems that involve both oscillatory and non-oscillatory dynamics.
- **Real-Time Implementation**: For real-world applications such as predictive maintenance and real-time system monitoring, implementing ASU-based DNNs in hardware and optimizing them for speed and efficiency will be critical. Ensuring that the network can process large volumes of data in real-time while maintaining high accuracy will be a major challenge.

The introduction of the **Amplifying Sine Unit (ASU)** in deep neural networks represents a significant step forward in modelling and predicting the behaviour of nonlinear oscillatory systems. The ASU enables the network to capture the periodic nature of these systems more effectively than traditional activation functions like ReLU, Sigmoid, and Tanh. By integrating this sine-based activation, the ASU-based DNN demonstrates superior performance in capturing complex oscillatory patterns, offering an efficient and robust approach to modelling nonlinear systems in engineering, physics, and other domains. While there are challenges to overcome, particularly related to computational complexity and model interpretability, the promising results suggest that this approach has wide-ranging applications and can be extended to various real-world scenarios.

CONCLUSION

This paper introduces an innovative deep neural network architecture based on the Amplifying Sine Unit (ASU), designed to improve the modelling of nonlinear oscillatory systems. The results from synthetic and real-world datasets demonstrate the superiority of the ASU-based network in capturing the oscillatory behaviour and nonlinearities of such systems. By offering a more accurate and efficient approach to system modelling, the ASU-based DNN has significant potential for a wide range of applications, including mechanical engineering, electrical systems, and signal processing.

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